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Robust Optimization of Fins by Taguchi Technique

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Abstract - "The sole aim of any design optimization technique is price and performance. In case of fins the optimization of price is concerned with minimum material requirement with improved temperature drop in terms of performance". In analytical or conventional optimization techniques, involved design parameters are related to each other in mathematical model, in form of ordinary/partial differential equation. If the design variables are such that they cannot be related mathematically then a mathematical model cannot be prepared and none of the classical design optimization techniques can be applied for solving the problem but Taguchi technique can account for such case. The application of mathematical optimization technique in case of fins involving parameters such as surface finish, effect of duct, bending, etc will be difficult to relate with the fins performance in a mathematical model. Over coming to this major limitation of classical techniques Taguchi method does not only account such variables but also provide robust optimization. Advantageously the method provides percent contribution of each variable/parameter for optimization of objective function.

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ROBUST OPTIMIZATION OF FINS BY TAGUCHI TECHNIQUE

Strictly as per the compliance and regulations of:

Robust Optimization of Fins by Taguchi Technique

Yash Mehta [α], Vimlesh Patel [α], Ms Priyanka Pathak [σ] & Dr. S.K. Dhagat [ρ]

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In analytical or conventional optimization techniques, involved design parameters are related to each other in mathematical model, in form of ordinary/partial differential equation.

If the design variables are such that they cannot be related mathematically then a mathematical model cannot be prepared and none of the classical design optimization techniques can be applied for solving the problem but Taguchi technique can account for such case.

The application of mathematical optimization technique in case of fins involving parameters such as surface finish, effect of duct, bending, etc will be difficult to relate with the fins performance in a mathematical model. Over coming to this major limitation of classical techniques Taguchi method does not only account such variables but also provide robust optimization. Advantageously the method provides percent contribution of each variable/parameter for optimization of objective function.

Taguchi techniques are Fractional Factorial experimental design techniques and use standard 'Orthogonal Arrays' of Fisher(1) for forming a matrix of experiments in such a way as to extract maximum important information with minimum number of experiments.

Economic performance of *fins* is not proportional to surface area but is proportional to effective surface area.

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I. Introduction

Taguchi built upon W. E. DEMING's(2),(3) observation that 85% of poor quality is attributed to the manufacturing process and only 15% to the worker. Quality and hence performance improvement start at very beginning. He proposed "Offline" strategies.

During 1940s Genichi Taguchi(4) has developed new statistical concepts of optimization tool which has been widely used in management and quality related optimization problem. In recent years it has been successfully implemented for the optimization of technical problems. Some of them are reported here:-

1. Optimization of preload on bolts (5)
2. Engine valve train noise study (2)
3. Study of crankshaft surface finish process(2)
4. Case study of Electrostatic powder coating process optimization(6)

Although the thermal optimization has been carried out conventionally (Hyung Suk Kang)(7) an attempt has been made here in implementation of Taguchi technique on optimization of fins.

Analysis of heat flow in the finned surface in conventional optimization is made on following assumptions(8),(9):-

- Temperature gradient over the cross sections is neglected and the heat transfer is treated as one- dimensional.
- Uniform heat transfer coefficient over the entire fin surface.
- Spacing between fins has no significant effect on heat dissipation rate.
- Negligible radiation exchange with the surrounding and other fins.
- Temperature gradient along the width remains constant.
- Perfect steady state heat dissipation.
- Material properties remain constant with the variation of temperature.

Since in Taguchi technique experiments are performed in ground situation hence 'Robust Optimization(10),(11)' is achieved eliminating the list of assumptions involve.

In analytical optimization it is assumed that rate of heat dissipation is double by doubling the number of fins but during practical observation such was not the case.

The expected cause is strong interaction(*) between different physical variables that has not been accounted in formulation of differential equation. But Taguchi technique provides percentage contribution of this interaction which may be some time more important than physical variables itself.

II. Problem Statement

a) Robust optimization of fins by Taguchi technique

Step1: conceptualization of the problem and formulation of measurable target quantity

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Conceptualization: - Following observations are made during formulation of problem statement.

1. The conventional fin (fig 1) has certain zone which shows insignificant temperature drop along with length that's appear in (fig 3) as center duct after removal of that zone.
2. Experimentally parabolic fins (fig2) are comparatively better than other shape for ratio of heat transfer to mass required. But the major limitation of such type of fins is that its performance is less than conventional fin for same length.
3. Contrary to theoretical parabolic it is not curved out exactly from the base (fig2) but away from the base to certain length (c) (fig4), experimentally better results were obtained.
4. Combination of above observations and redefining the parabola yields fins of (fig 4) which has been further optimized by Taguchi technique for close tolerance.

Here the temperature drop (ΔT) is measurable target quantity which is difference between temperatures at base and tip of fin.

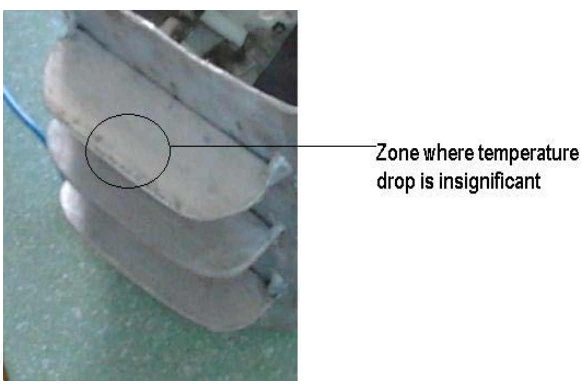


fig1

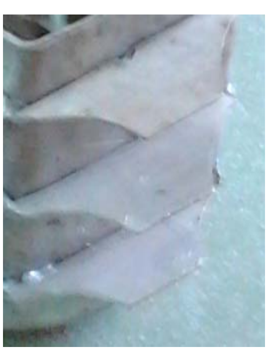


fig2

fig3

∗Interaction –The mutual action between physical variables that may give entirely different resultant then expected one, if both are not fully independent.

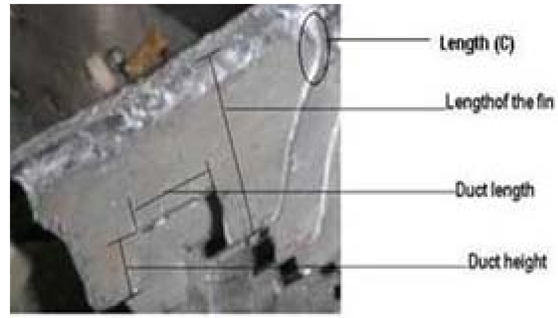


fig 4

Step2: Selection of parameters

Table 1.1 : Planning and Carrying Out Experiments:-

Factor No.	Factor Specification	Factor Level1	Factor Level2
1	B : BEND	B1 : SIMPLE	B2 : BEND
2	L : LENGTH	L1 : 36mm	L2 : 30 mm
3	L × B	-------	------
4	H:DUCT HEIGHT	H1 : 10mm	H2 : 15 mm
5	N : No. of FINS	N1 : 7	N2 : 4
6	L × H	-------	------
7	l : DUCT LENGTH	l1 : 26 mm	l2: 34

Fig 5

Step 3: In full factorial experimentation with 7 parameters and 2 levels of each would require $2^7 = 128$ number of experiments. But applying Dr. Taguchi's L_8 series only 8 set of experiments are required. And hence maximum information is extracted from minimum number of experiments.

Table 1.2 : L_8 (2^7) Series with allocated interaction(*)

Experiment	Factor								
	Bend (B)	Length(L)	B x L interaction	Height(H)	No. Of Fins(N)	L x H interaction	Duct Length	Result (Temperature drop °C)	Target (Y_i)
Trial 1	1	1	1	1	1	1	1	3.05	305
Trial 2	1	1	1	2	2	2	2	3.3	330
Trial 3	1	2	2	1	1	2	2	2.5	250
Trial 4	1	2	2	2	2	1	1	3.10	310
Trial 5	2	1	2	1	2	1	2	3.63	363
Trial 6	2	1	2	2	1	2	1	2.94	294
Trial 7	2	2	1	1	2	2	1	2.9	290
Trial 8	2	2	1	2	1	1	2	2.21	221
								Total(∑Y)	=2363

Step4: Calculation of average effect of parameters:-

$$\sum B1 = \frac{305+330+250+310}{4} = 298.75$$

$$\sum (B\times L)_1 = \frac{305+330+290+221}{4} = 286.5$$

Similarly, effect of other parameters are tabulated in table 1.2

$\sum B2 = 292$ $\sum L1 = 323$ $\sum L2 = 267.75$

$\sum N1 = 267.5$ $\sum N2 = =323.25$ $\sum (L\times H)_1 = 299.75$

$\sum (L\times H)_2 = 291$, $\sum l1 = 299.75$, $\sum l2 = 291$, $\sum (B\ X\ L)_2 = 304.25$

Table 1.2 : Main Effect Table

Serial No.	Main Effect	Level 1(Le_1)	Level 2(Le_2)	Le_2-Le_1
1	B	298.75	292	-6.75
2	L	323	267.75	-55.25
3	B × L	286.5	304.25	17.75
4	H	302	288.75	-13.25
5	N	267.5	323.25	55.75
6	L × H	299.75	291	-8.75
7	L	299.75	291	-8.75

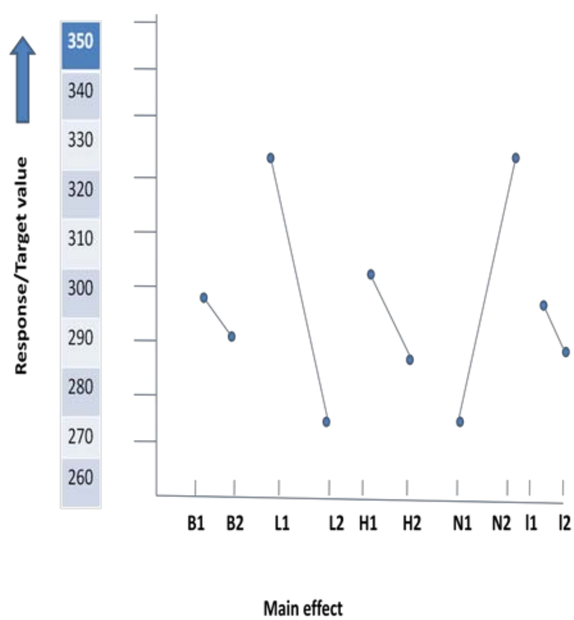


Fig 6

Average effect of Interaction:-

$B_1 L_1 = (305+330)/2 = 317.5$

(*) Allocation of interaction:-Dr. Taguchi has already defined the rules forfff for positioning the interactions and common variable (here L)

Similarly,

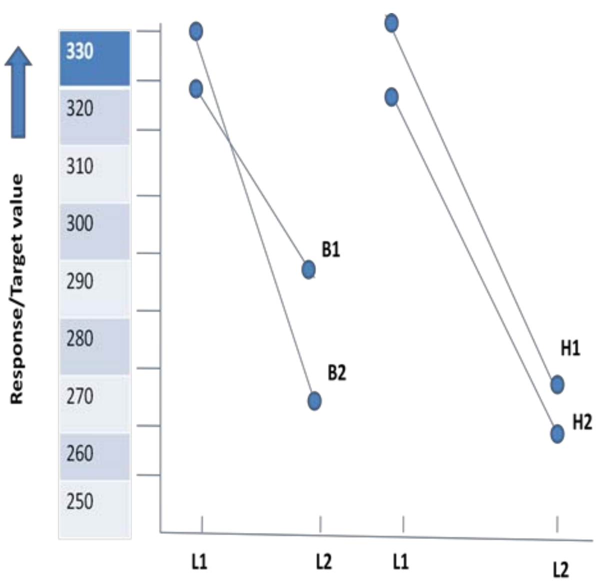
$H_1 L_1 = 334$

$B_1 L_2 = 280$ $H_2 L_1 = 312$

$B_2 L_1 = 328.5$ $H_1 L_2 = 270$

$B_2 L_2 = 255.5$ $H_2 L_2 = 265.5$

The interaction diagram below indicates there exist strong interaction between bend (B) and length (L).Whereas the interaction between duct height (H) and length (L) is limited or weak.



Interaction

fig7

Step 5 : ANOVA (Analysis of variance)

Step 5.1 : Total of all result:-

$$\sum Y = T = 2363$$

Step 5.2 : Procedure of ANOVA

$\mathbf{CF} = \frac{T^2}{n}$ where n is the number of experiments

$$CF = \frac{2363^2}{8} = 637971.125$$

*Step 5.3 :*Total sum of Square

$$S_T = \sum_{i=1}^{8} Y_i^2 - CF$$

$$S_T = (305^2+330^2+250^2+310^2+363^2+294^2+290^2+221^2)-(697971.125)$$

$$S_T = 13699.875$$

Step 5. 4 : Factor sum of square

$$S_B = \frac{(\sum B_1)^2}{n_{A1}} + \frac{(\sum B_2)^2}{n_{A2}} - CF$$

$$S_B = \frac{(1195)^2}{4} + \frac{(1168)^2}{4} - 697971.125 = 91.125$$

$$S_{B \times L} = \frac{(1146)^2}{4} + \frac{(1217)^2}{4} - 697971.125 = 630.125$$

Similarly,

$S_L = 6105.125$ $S_H = 351.125$ $S_N = 6216.125$

$S_{H \times L} = 153.125$ $S_I = 153.125$

Step 5.5 : Total and Factor degree of Freedom

DOF total = No. of Experiment – 1

$$f_T = n-1 = 8-1 = 7$$

$$f_B = \text{No. of Level} -1 = 2-1 = 1$$

Similarly,

$$f_L = f_H = f_N = f_I = 2-1 = 1$$

$$f_{H \times L} = 1 \times 1 \qquad f_{B \times L} = 1 \times 1$$

Degree of error term up till $f_e = f_T - (f_B + f_L + f_H + f_N + f_{H \times L} + f_{B \times L} + f_I) = 7-7 = 0$

With the error degree of freedom equal to zero,

$$f_e = 0.$$

Information regarding the error sum of square cannot be determined. In addition F ratios for factor cannot be calculated since the calculations involve f_e ,(pooled) to form a new nonzero estimate of the error term.

Step 5.6 : Mean square (variance)

$$V_B = \frac{S_B}{f_B} = \frac{91.125}{1} = 91.125 \qquad V_{B \times L} = \frac{S_{B \times L}}{f_{B \times L}} = 630.125$$

Similarly,

$V_L = 6105.125$ $V_H = 351.125$ $V_N = 6216.125$

$V_{H \times L} = 153.125$ $V_I = 153.125$

$V_e = \frac{S_e}{f_e} = \frac{0}{0}$ = indeterminate form.

As the variance of error term V_e is zero. The Variance ratio and pure Sum of Square S' cannot be calculated. Following method is adapted to recalculated percentage contribution.

Step 5.7 : Intial percentage contribution :-

$$P_B = \frac{S_B}{S_T} = \frac{91.125}{697971.125} = 0.66\%$$

$$P_{B\,X\,L} = \frac{S_{B\,X\,L}}{S_T} = \frac{630.125}{697971.125}\ 4.6\%$$

$P_L = 44.56\%$ $P_H = 2.56\%$

$P_{H\,X\,L}1.11\%$ $P_N = 45.37\%$ $P_l = 1.11\%$

P_e cannot be calculated since V_e is zero.

ANOVA Table 1.4

Column	Factor	*f*	S	v	P
1	Factor B	1	91.125	91.125	0.66%
2	Factor L	1	6105.125	6105.125	44.56%
3	Interaction BXL	1	630.125	630.125	4.6%
4	Factor H	1	351.125	351.125	2.56%
5	Factor N	1	6216.125	6216.125	45.37%
6	Interaction BXH	1	153.125	153.125	1.11%
7	Factor l	1	153.125	153.125	1.11%
All other error	0	0	0	0	100%
Total			13699.875		

Step 6 : Pooling (*) :-

The effect of factor B is less than unity(0.66% only), Hence this factor is pooled to obtained non zero Estimates of S_e and f_e.

Step 7 Sum of Square of Error :

Let , $S_e = S_T - (S_L + S_{B\,X\,L} + S_H + S_N + S_{H\,X\,L} + S_l)$

$= 13699.875 -$

$(6105.125+630.125+351.125+6216.125+153.125+153.125)$

$S_e = 91.125$

Degree of freedom of error term

$f_e = f_T - (f_L + f_{B\,X\,L} + f_H + f_N + f_{H\,X\,L} + f_l)$

$= 7-(1+1+1+1+1+1)$, $f_e = 1$

Variance of error term

$$V_e^{(\Delta)} = = \frac{S_e}{f_e} = \frac{91.125}{1} = 91.125$$

Step 8 : F ratio of significant factors

$$F_L = \frac{V_L}{V_e} = \frac{6105.125}{91.125} = 66.99$$

$$F_{B\,x\,L} = \frac{V_{B\,x\,L}}{V_e} = \frac{630.125}{91.125} = 6.914$$

$F_H = 3.85$ $F_N = 68.21$ $F_{Hx\,L} = 1.68$

$$F_l = \frac{V_l}{V_e} = \frac{153.125}{91.125} = 1.68$$

Pure Sum of Square S', for significant figure

$$S'_L = S_L - (V_e \ X\ f_L)$$

$=6105.125-(91.125 \times 1) =6014$

(*)Pooling – pooling means elimination of factors having insignificant % contribution.

(Δ) Compare the new variance and new percentage contribution of error term with the results of without pooling

Similarly,

$S'_{B\,x\,L} = 539$ $S'_{H} = 260$ $S'_{N} = 6125$ $S'_{H\,x\,L} = 62$ $S'_{l} = 62$

Step 9 : New Percent contribution

$$P_L = \frac{S'_L}{S_T} = \frac{6014}{13699.875} = 43.9\%$$

$$P_{B\,x\,L} = \frac{S'_{B\,x\,L}}{S_T} = \frac{539}{13699.875} = 3.93\%$$

$P_H = 1.9\%$ $P_N = 44.7\%$ $P_{H\,x\,L} = 0.45\%$ $P_l = 0.45\%$

$$P_e^{\Delta} = 100\%-(43.9+3.93+1.9+44.7+0.45+0.45)$$

$$=4.67\%$$

From F Table .find the F valve at

n_1 = DOF of factor L = 1

n_2 = DOF of Error term=1

At a confidence level of 90% (confidence level)

F_L = 39.864 (from F table)

As F_L from experiment (66.99) is larger than F Table

Value (39.864) the factor L is not needed to be pooled.

Pooled ANOVA Table 1.5

Step10 : Estimated Result at Optimum condition

Pooled factor are not included in the estimate.

Grand average performance: $T = \frac{2363}{8} = 295.375$

As the Factor L, BXL, H, HXL, N, l are significant

$$= \bar{T} + (\bar{L}1-\bar{T})+(\overline{B\ X\ L})_2-\bar{T}+(\bar{H}1-\bar{T})+(\bar{N}2-\bar{T})+(\overline{B\ X\ L})_1-\bar{T})+(\bar{l}1-\bar{T})$$

$$=295.375+(323-295.375)+(304.5-295.375)+(302-295.375)+(323.25-295.375)+(299.75-295.375)+(299.75-295.375)$$

$$=375.125$$

III. Conclusion

- Note the optimum condition for the "higher the better"
  Configuration i.e. higher is the temperature drop better is the performance.
  Following are the opted specifications: -

  B_2 bend
  L_1 36 mm
  H_1 10mm
  N_2 4
  l_1 26mm

- From the calculation the percentage contribution of duct length (l) is less than unity hence this particular parameter can be adjusted according to economic consideration without affecting the performance to larger extent, hence opt l_2.
- In theoretical problems the performance of single fin (in terms of rate of heat dissipation) is multiplied with number N (where N is number of fins involved), to get the cumulative performance of N number of fins. But the experiment suggest so is not the case, because N as a parameter has only 78% contribution in the cumulative result. (Shown in fig).
- The duct and the shape have peculiar effect (cornering effect) in the governing of heat dissipation, in which even after reducing the surface area in (certain zone) comparable performance has been achieved. In this particular experiment the one fact reveled is percentage contribution of interaction is more than individual parameter (for bending) and but it is difficult to point out this effect in conventional method.
- (a) If 4 conventional fins are compared with 4 optimized fins by this experiment then 22 % cost saving can be achieved with improved performance.
  (b) if performance of 7 conventional fins is compared with 4 optimized fins them primary calculation indicate that at the expense of 32% of the performance up to 55% cost can be saved.

The pie chart shown below represents the percentage contribution of each parameter including interactions on temperature drop.

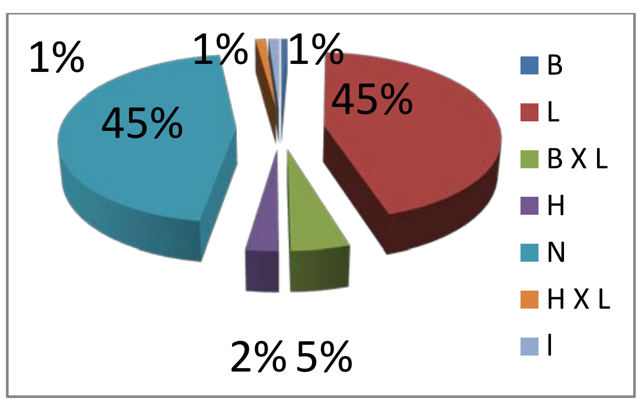


Fig 8

IV. Scope

With more number of parameters, (including more number of possible interactions) and higher series of orthogonal arrays such as L_{16} (4^5),L_{18} (2^1 x 3^7), L_{27} (3^{13}), L_{32} (2^1 x 4^9) series, further refinement can be carried out.

Advantages of the Taguchi techniques

1) The Taguchi techniques are applicable for both type problems for which mathematical modeling is possible and for which formation of ordinary/partial differential equation is not possible.
2) Advantageously over full factorial experimental optimization techniques maximum information is extracted with minimum number of experiments.
3) Mathematical modeling is not required.
4) Being a fractional factorial method and use of orthogonal arrays ensures minimum time and minimum cost of experimentation.

Disadvantages of the Taguchi techniques

1) All the possible interactions amongst the parameters cannot be studied as this technique is not based on full factorial method.
2) Number of levels for the design variables and parameters are required to be assumed to capture the true nature of variation of the variables and parameters.
3) Positioning hinders the repetition of experiment involving interaction.

V. Abbreviation & Symbols

- B, L, H - variable used in design of an experiments
- C.F.- Correction factor
- e- experimental error
- f, n- Degree of freedom(DOF)
- F- variance ratio
- N- The number of experiments
- P-The percent contribution of variable
- S- The sum of squares
- S'- The net/pure sum of squares
- T- The sum of observations
- V- the variance (mean square ,s/f)
- Y- Result measured in terms of quality characteristics example Height, duct length, length, etc.
- V_e- Variance of error terms

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11. C. Zang a, M.I. Friswell a, J.E. Mottershead ba 'A review of robust optimal design and its application in dynamics', Department of Aerospace Engineering, University of Bristol, Queen's Building, Bristol BS8 1TR, United Kingdom Department of Engineering, University of Liverpool, Brownlow Hill, L United Kingdom Received &accepted 13 November, 2003 -04.